15/10/2009

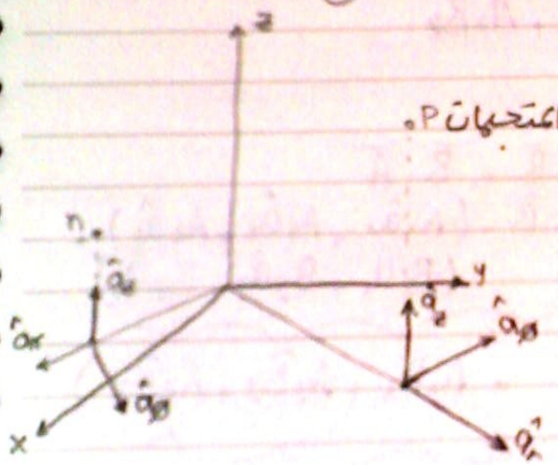
sheet(1)

$$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

$$\bar{A} = A_r \hat{a}_r + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

$$\bar{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$$

• اتجاهات المحاور الكرتيزيه ثابته مهما اختلف موقع النقطه على العكس من المحاور الاسطوانيه او الكرويه



• تحويل النقطه يختلف عن تحويل المتجهات P.

← تحويل النقطه

$$r = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1} \frac{y}{x} \quad ; \quad z = z$$

← تحويل المتجهات

	a_r	a_ϕ	a_z
a_x	$\cos\phi$	$-\sin\phi$	0
a_y	$\sin\phi$	$\cos\phi$	0
a_z	0	0	1

$$a_x = \cos\phi \, \hat{a}_r - \sin\phi \, \hat{a}_\phi$$

$$a_y = \sin\phi \, \hat{a}_r + \cos\phi \, \hat{a}_\phi$$

← العوض بيهم ف المعادله

① $|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$= \sqrt{A_r^2 + A_\phi^2 + A_z^2}$

بيعبر عن مسافة

② $\bar{A} = |\bar{A}|\,\hat{a}_A \rightarrow \hat{a}_A = \dfrac{\bar{A}}{|A|}$

③ $\bar{A}\cdot\bar{B} = A_xB_x + A_yB_y + A_zB_z$

→ Dot product :-

- $\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}$
- $\bar{A}\cdot\bar{B} = (A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z)\cdot(B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z)$

$$\bar{A}\cdot\bar{B} = A_xB_x + A_yB_y + A_zB_z$$

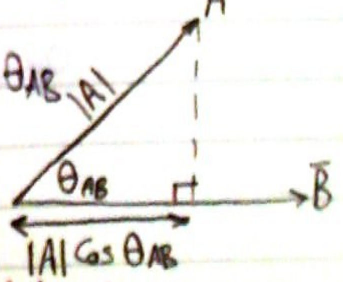
- $\bar{A}\cdot\bar{B} = |A||B|\cos\theta_{AB}$

$\theta_{AB} = \cos^{-1}\dfrac{\bar{A}\cdot\bar{B}}{|A||B|}$

- projection of $\bar{A}$ on $\bar{B}$

$= |A|\cos\theta_{AB}$

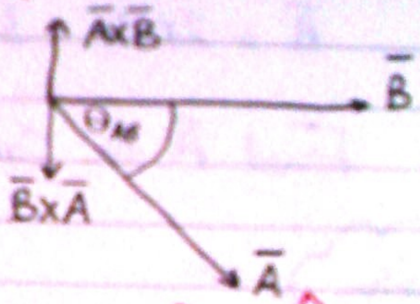
$= \dfrac{\bar{A}\cdot\bar{B}}{|B|}$



* الضرب المقداري (Dot product) يستخدم بنفس الخواص من كل الاحداثيات

→ Cross product

• $\bar{A}\times\bar{B} = -(\bar{B}\times\bar{A})$



• $\bar{A}\times\bar{B} = |A||B|\sin\theta_{AB}\,\hat{a}_n$

unit vector normal to The plane contains $\bar{A}$ & $\bar{B}$

• $\bar{A}\times\bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

• $\theta_{AB} = \sin^{-1} \dfrac{|\bar{A}\times\bar{B}|}{|A||B|}$

sheet (1)

prob (4)

$\bar{A} = 2r\hat{a}_r - 3r\sin\phi\,\hat{a}_\theta$

Find $\bar{A}$ in Cartesian Co.ord

Soln

ملحوظة

الفرق بين الـ vector والـ (vector field)

* Vector → constant vector at any point

i.e $\bar{A} = 3\hat{a}_x + 2\hat{a}_y + \hat{a}_z$

** vector field → change its direction at each point

i.e $\bar{A} = 3x\,\hat{a}_x + 2y\,\hat{a}_y + z\,\hat{a}_z$

$\hat{a}_r = \sin\theta\cos\phi\,\hat{a}_x + \sin\theta\sin\phi\,\hat{a}_y + \cos\theta\,\hat{a}_z$

$\hat{a}_\theta = \cos\theta\cos\phi\,\hat{a}_x + \cos\theta\sin\phi\,\hat{a}_y - \sin\theta\,\hat{a}_z$

$\therefore \bar{A} = (2r\sin\theta\cos\phi - 3r\sin\phi\cos\theta\cos\phi)a_x + \ldots\ldots$

→ we know that :-

$x = r\sin\theta\cos\phi \quad ; \quad y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$\bullet\ \bar{A}_x = \left(2x - 3z\,\dfrac{x}{r\sin\theta}\cdot\dfrac{y}{r\sin\theta}\right)\hat{a}_x$

$= \left(2x - \dfrac{3xyz}{r^2\sin^2\theta}\right)\hat{a}_x$

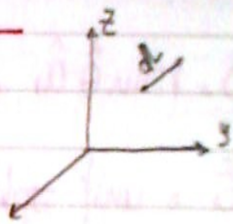
$$\bar{A}_x = \left(2x - \frac{3xyz}{x^2+y^2}\right) a_x$$

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

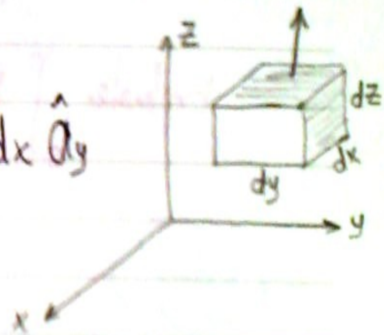
$$x^2+y^2 = r^2\sin^2\theta$$

Differential elements :-



① Cartesian :

$$\bar{dL} = dx\, a_x + dy\, \hat{a}_y + dz\, \hat{a}_z$$



$$\bar{ds} = dx\,dy\, \hat{a}_z + dy\,dz\, \hat{a}_x + dz\,dx\, \hat{a}_y$$

$$dV = dx\,dy\,dz$$

② Cylindrical :-

$$\bar{dL} = dr\, \hat{a}_r + r\,d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$$

$$\bar{ds} = r\,d\phi\,dz\, \hat{a}_r + dr\,dz\, \hat{a}_\phi + r\,dr\,d\phi\, \hat{a}_z$$

$$dv = r\,dr\,d\phi\,dz$$

③ Spherical

$\overline{dL} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$

$\overline{ds} = r^2\sin\theta\,d\theta\,\hat{a}_r + r\sin\theta\,dr\,d\phi\,\hat{a}_\theta + r\,dr\,d\theta\,\hat{a}_\phi$

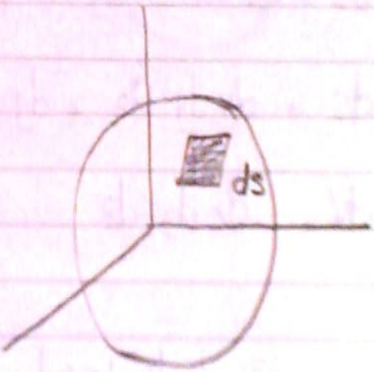
$dv = r^2\sin\theta\,dr\,d\theta\,d\phi$

Sh (1) pb (6)

$\overline{D} = r\sin\theta\,\hat{a}_r + r\sin\theta\,\hat{a}_\theta$

$\overline{S}$ = unit sphere centered at origin

Evaluate $\int_S \overline{D}\cdot\overline{ds}$



notes

• الدايره قطرها ثابت

$\therefore r = 1 \qquad dr = 0$

$\therefore \overline{ds} = \sin\theta\,d\theta\,\hat{a}_r$

← من الاخر خد الكومبوننت اللى ف اتجاه الثابت بس

ملحوظه = لو ال (θ) وال (r) ثابتين يبقى انا بدور فى حلقه مفرغه

يعنى ملهاش مساحه يعنى $dS = 0$

Soln

$$\int_S \overline{D}\cdot\overline{ds} = \iint (r\sin\theta)(r^2\sin\theta)\,d\phi\,d\theta$$

$$= \int_0^{\pi}\int_0^{2\pi} \sin^2\theta\,d\phi\,d\theta$$

$$= \int_0^{\pi} \phi \Big|_0^{2\pi} \sin^2\theta \, d\theta = 2\pi \int_0^{\pi} \sin^2\theta \, d\theta$$

$$= 2\pi \int_0^{\pi} \frac{1}{2} \left(1 - \cos 2\theta\right) d\theta$$

$$\int_S \bar{D} \cdot \bar{ds} = \pi^2$$

Del operator (Nabla)

$$\bar{\nabla} = \frac{\partial}{\partial x} \hat{a}_x + \frac{\partial}{\partial y} \hat{a}_y + \frac{\partial}{\partial z} \hat{a}_z$$

1. Grad :- (Scaler → vector)

$$\bar{\nabla} V = \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z$$

2. Divergence :- (vector → Scaler)

$$\bar{\nabla} \cdot \bar{F} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$$

3. Curl :- (vector → Vector)

$$\bar{\nabla} \times \bar{F} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

4.) Laplacian :- (scaler $\rightarrow$ scaler)

$$(\bar{\nabla})^2 V = \bar{\nabla} \cdot \bar{\nabla} V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$\Rightarrow$ $\bar{\nabla} \times \bar{\nabla} F$ = Curl (grad) = 0

$\Rightarrow$ $\bar{\nabla} \cdot (\bar{\nabla} \times \bar{F})$ = div (Curl) = 0